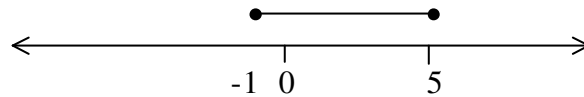


3AMAS Exam revision

Ch 1: Absolute value

Question 1. [10 marks]

The solution to the inequality $|2x - 1| \ ? \ |x + a|$ shown below is defined such that $a > 0$,



- (a) Find the value for a . [Complex; either] [4 Marks]
- (b) Replace the $?$ by the appropriate inequality ($<$, $>$, etc.) to make the statement represent the solution. [Simple; either] [2 Marks]
- (c) Solve the inequality $|x^2 - x - 21| < 39$. Clearly showing the method used [Complex/Resource Rich] [4 Marks]

Question 2. (7 marks)

Determine the solution set for $|2x + 3| \leq |3x + 7|$ [Complex; Resource free]

Question 3. (7 marks)

Solve the inequality $|x + 2| < \frac{1}{3}x + 1$ [Complex; Resource free]

Question 4. (6 marks)

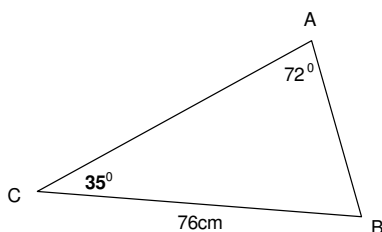
Find a and b given that $|x - a| = |x - 2| - 5$ has as its solution set $\{x \in \mathbf{R} : x \geq b\}$ [Complex, either]

Ch 2: Trigonometry; Ch 5: Radian Measure

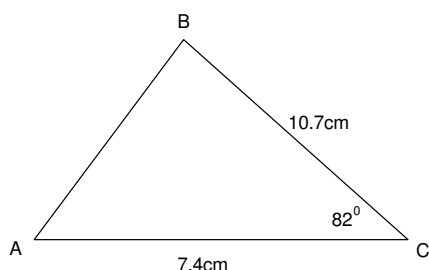
Question 5. [2, 2 = 4 marks]

Reminder: make sure your calculator is set to work appropriately with degrees or radians.

- (a) Find the length of \overline{AC} in the following triangle: [Simple; Resource rich]

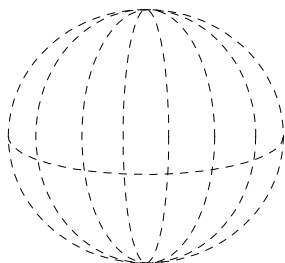


- (b) Calculate the area of $\triangle ABC$: *[Simple; Resource rich]*



Question 6. [6 marks]

Calculate the total distance travelled by an aeroplane (to the nearest kilometre) which travels without stopping from Perth (32°S , 116°E) due north to 北京 (Beijing) (38°N , 116°E) and then due west to Αθήνα (Athens) (38°N , 24°E). (Assume that the plane flies at a negligible height above the surface of the earth and that the radius of the Earth is 6350 km). *[Simple; Resource rich]*



Question 7. [3, 2, 2, 2 = 9 marks] [Resource Rich; Complex]

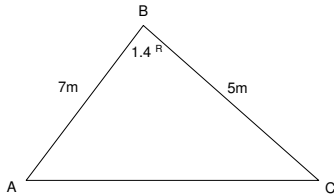
An observer in a fishing boat looks due north towards a lighthouse as the deckhands set craypots at point B. Point B is 300m from point A at sea level directly below the lighthouse. The angle of elevation of the lighthouse from the observer is 19° . Some time later, the fishing boat is at a point C, bearing 150° from A, where the angle of elevation of the lighthouse from the boat is 14° .

- Draw a single diagram to represent the situation labelling points A, B and C and including all relevant angles and distances.
- Find the height of the lighthouse above sea level.
- How far is C from A?
- How far did the boat travel from B to C?

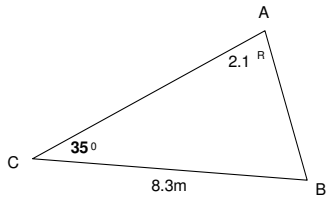
Question 8. [1, 1, 4 and 3 = 9 marks] *[Simple; Resource Rich]*

Find the length of \overline{AC} in the following triangles:

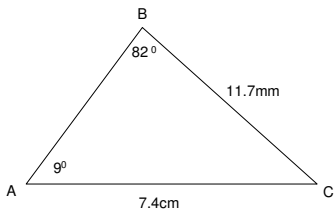
(a)



(b)

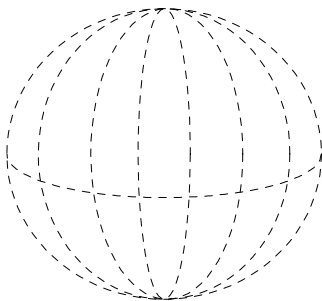


(c) Calculate the area of $\triangle ABC$:



Question 9. [4 marks] *[Simple; Resource rich]*

Find the shortest distance, along the parallel of latitude, surface of the earth from Milano (45°N , 9°E) to Ottawa (45°N , 76°W) if the radius of the earth is 6350 km.



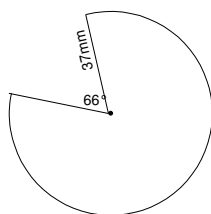
Question 10. [3, 2, 5 and 4 = 14 marks] [Complex; Resource rich]

An observer stands atop a vertical cliff and looks due North out to sea. She notices a fishing boat tied to a buoy B, 600m from A (the cliff base). The angle of depression of the boat from the observer is 58° . Some time later, the fishing boat is at a point C, bearing 140° from A, where the angle of elevation of the observer from the boat is 24° .

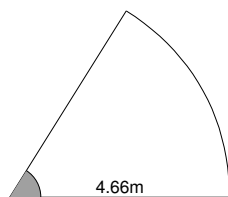
- Draw a single diagram to represent the situation labelling points A, B and C and including all relevant angles and distances.
- Find the height of the cliff.
- How far did the boat travel directly from B to C?
- Find $\angle ABC$ and hence determine the bearing of C from B.

Question 11. [3,2 marks] [Simple; Resource rich]

- Determine the perimeter to the nearest mm:



- Determine the angle to the nearest degree given that the arc length is 5.54m:

**Question 12. [2, 2, 2, 3 = 9 marks] [Simple; Resource free]**

Simplify the following, giving answers with rational denominators where appropriate. All answers are to be expressed as exact values.

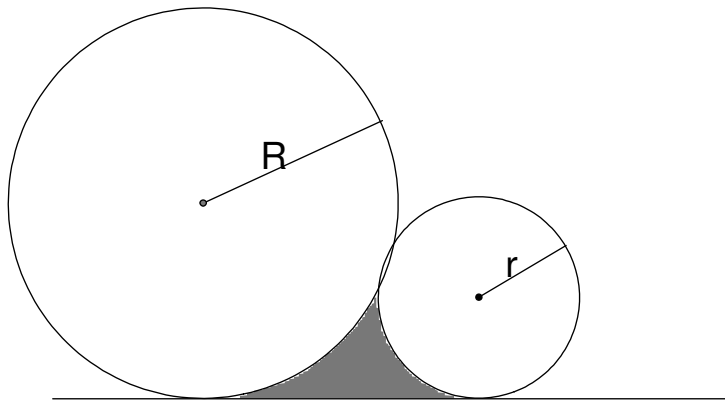
- $(6\sqrt{2} - 4\sqrt{3})(2\sqrt{3} + 3\sqrt{2})$
- $\sqrt{48} \times \sqrt{15}$
- $\frac{3}{4\sqrt{5}}$
- $\frac{\sqrt{7} - 1}{\sqrt{7} + 1}$

Question 13. [1,2=3 marks]

Give the exact value of the following (with rational denominators where appropriate):

- $\tan 150^\circ$
- $\frac{\cos 120^\circ}{\sin 225^\circ}$

Question 14. [Complex; resource free or resource rich]



Find the shaded area in terms of R and r .

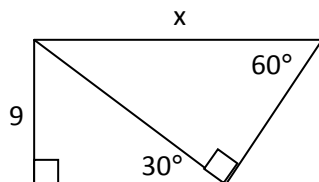
Question 15. [3 marks]

Find the smallest angle of a triangle with sides of length 23 cm, 30 cm and 33 cm. (Give your answer correct to 2 decimal places.)

Question 16. [2, 2, 3 marks]

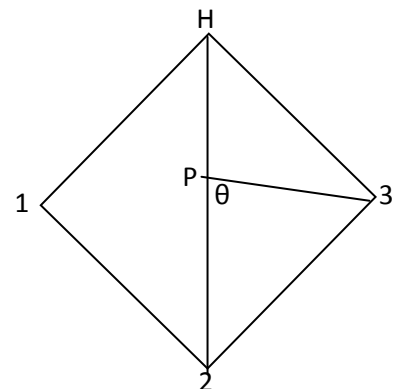
Give the **exact** value of:

- (a) (i) $\cos 135^\circ$
(ii) $\tan 210^\circ$
- (b) $\sin^2 45^\circ + \cos^2 120^\circ$
- (c) x in the following diagram.



Question 17. [3, 3 marks]

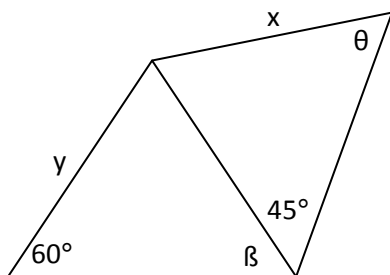
A baseball diamond is a square of side 27.4 m with bases 1, 2, 3 and Home, H, as marked on the diagram below. The pitcher's mound P is on the diagonal $H2$, such that the distance HP is 18.4 metres. (Give your answers correct to 2 decimal places.)



- (a) Find the distance $P1$, the distance from P to 1st base.
- (b) Find the angle θ as marked on the diagram.

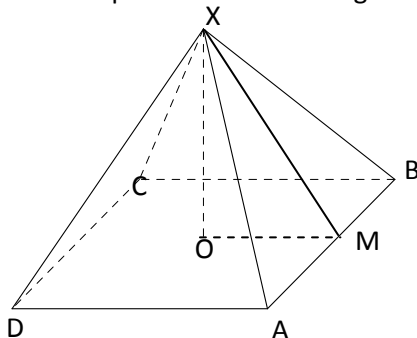
Question 18. [4 marks]

For the given diagram, express y **exactly** in terms of x , θ and β .



Question 19. [2, 2, 2 marks]

The square based pyramid below has base ABCD, with point X directly above point O, the centre of the base. Point M is the midpoint of AB. The length AB = 50 cm and OX = 60 cm.

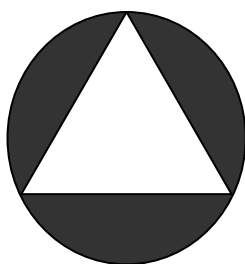


Find the following (correct to 2 decimal places):

- length XB.
- size of $\angle XBA$
- size of $\angle XMO$

Question 20. [5 marks]

An equilateral triangle is circumscribed by a circle of radius 10 cm. Find the shaded area correct to 2 decimal places.



Question 21. [2, 2, 2 marks]

A yacht sails from a harbour for 12 km on bearing 075° , and then for a further 6 km on bearing 340° . (Give answers correct to 2 decimal places.)

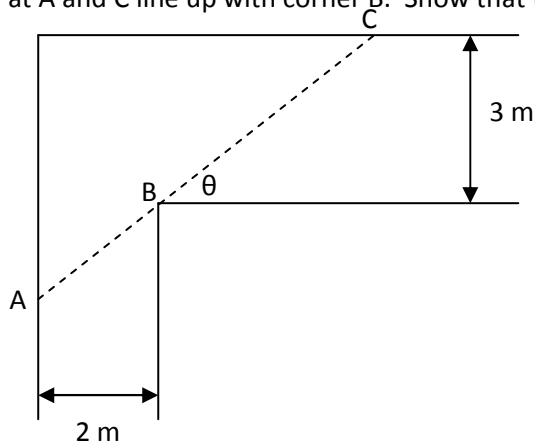
- Draw a diagram to show the path taken by the yacht.
- Find the distance the yacht is now from the harbour.
- Find the bearing on which the yacht needs to sail to return to the harbour.

Question 22. [4 marks]

Find the rotational speed in km/hour of the surface of the Earth at Perth, located at 32°S , 116°E . Use the radius of the Earth = 6350 km. (Give your answer correct to the nearest 10 km/hour.)

Question 23. [4 marks]

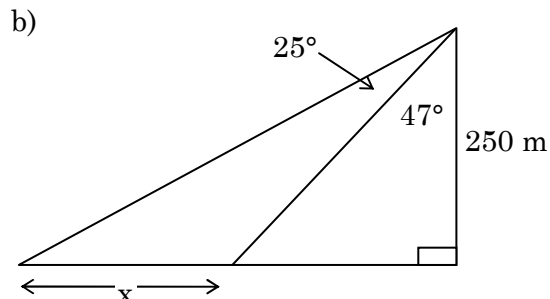
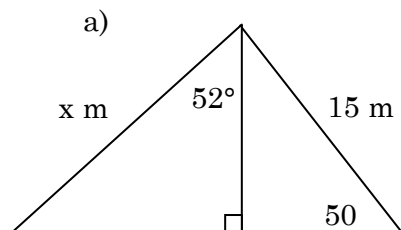
Two footpaths meet at right angles and are 3 m wide and 2 m wide respectively. It is noticed that signposts at A and C line up with corner B. Show that the distance between A and C is given by:



$$\frac{2}{\cos \theta} + \frac{3}{\sin \theta}$$

Question 24.

Find the value of x in each of the following using trig ratios;



Question 25. [4 marks]

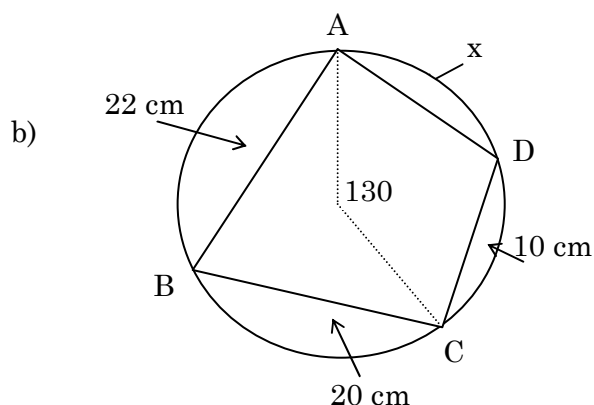
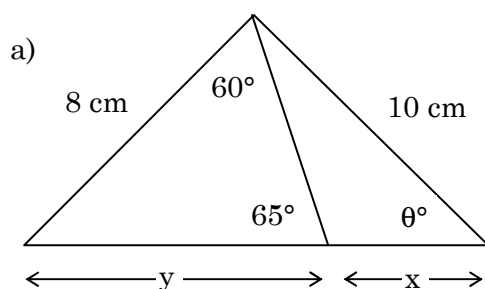
The pyramids of Egypt have square bases. The largest, that of Cleops, has a base of length 232.9 metres and its height is 146.3 metres. Find the angle between a sloping face and horizontal base.

Question 26. [7 marks]

A plane flies without stopping from A (30°N , 120°E) due west to B (30°N , 20°W) and then due south to C (20°S , 20°W). If it flies at a negligible height above the earth's surface at a cruising speed of 700km/h, how long will the flight take? Compare this with the same the time taken if the speed is the same, but the plane flies at a height of 10km. Assume the earth's radius is 6370km.

Question 27.

Find the pronumerals in each of the following:



Question 28.

Prove $a^2 + b^2 + c^2 = 2(bc \cos A + ac \cos B + ab \cos C)$ in any $\triangle ABC$.

Question 29.

Find the exact values for the following, simplify where possible.

a) $\sqrt{32}$

b) $\sqrt{(10^2 - 5^2)}$

c) $\sqrt{24} \times \sqrt{6}$

d) f) $\frac{\sqrt{6}}{\sqrt{10} - \sqrt{6}}$

Question 30.

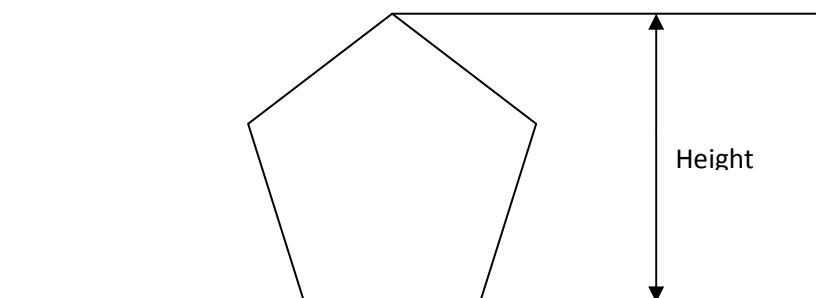
A rectangle, PQRS has PQ = 10cm and PS = $5\sqrt{3}$ cm. If T, the midpoint of PQ, is joined to S, find the sizes of the two angles formed at S. Prove that TSR is an equilateral triangle.

Question 31. [5 marks]

In $\triangle ABC$, $a = 7$ cm, $b = 9$ cm and $A = 50^\circ$. Find the size(s) of B, C and c (correct to one decimal place).

Question 32. [4 marks]

A regular pentagon with sides of length 5cm is upright on a horizontal surface as shown below. Find the height of the top vertex of the pentagon above the surface.



Question 33. [4 marks]

A ship sails 1200 km due west. If the longitude changes by 32° , find the latitude of the ship. [Radius of the earth = 6350km]

Question 34. [6 mark]

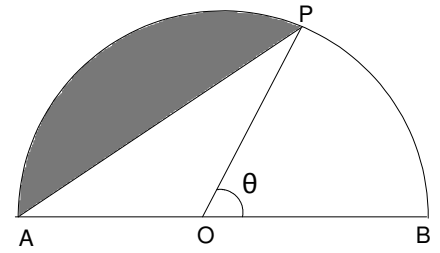
Two fire lookout towers A and B are situated in thick forest. A is 10 km due east of B and the nearest town is located 15 km from A on bearing 035° . Smoke from a fire is observed on bearing 022° from B, and on bearing 349° from A.

- (a) How far is the fire from the town, and
- (b) in which direction should firefighters be sent from the town?

Question 35. [Complex; Resource free]

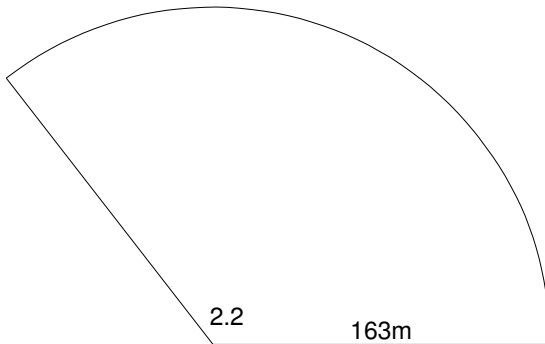
The diagram shows a semi-circle having AB as its diameter. Given angle POB = θ

- i) State the size of angle AOP
- ii) Hence find in terms of θ the area of the shaded region.

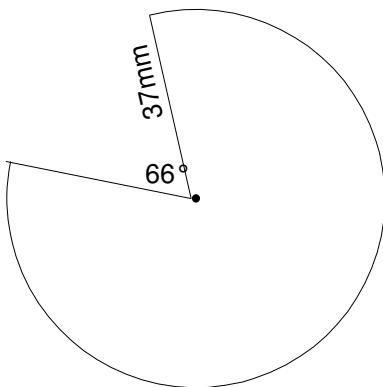


Question 36. [1, 3, 2, 3 = 9 marks]

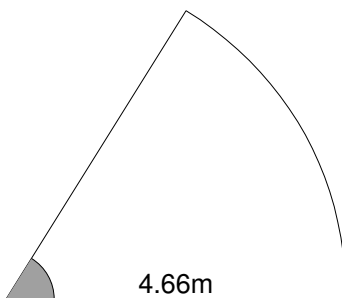
- (a) Determine the arc length to the nearest metre:



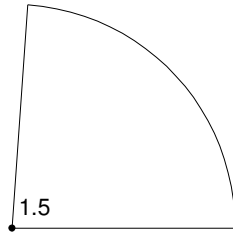
- (b) Determine the perimeter to the nearest mm:



- (c) Determine the angle to the nearest degree given that the arc length is 5.54m:



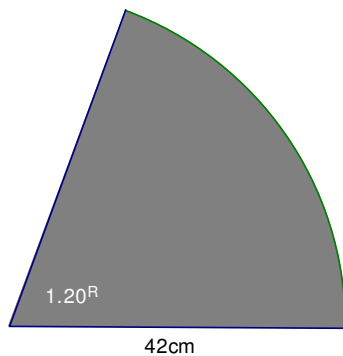
- (d) Determine the radius to the nearest centimetre given that the perimeter of the shape is exactly 10 metres and the central angle is 1.5 radians:



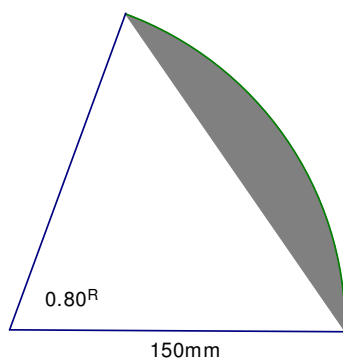
Question 37. [2, 3 = 5 marks]

Determine the area of the shaded region.

(a)



(b)



Question 38. [2 marks]

Convert 65° to radians exactly.

Question 39. (2+3=5 MARKS)

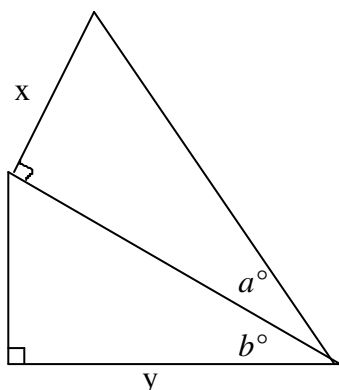
Without using a calculator (hence show working) , give the exact value of :

a) $\tan\left(\frac{5\pi}{3}\right)$

b) $\sin\left(\frac{-3\pi}{4}\right) - \cos\left(\frac{13\pi}{4}\right)$

Question 40. (4 MARKS)

For the following, find y in terms of x , a and b



Question 41. (4 MARKS)

Triangle ABC has angle $B = 40^\circ$, $b = 8$ cm and $c = 11$ cm. Solve the triangle for all sides and angles.

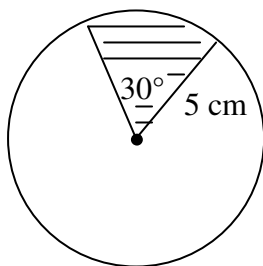
Question 42. (3 MARKS)

A sector of a circle of a radius 10 cm has a perimeter of 40 cm. Find the area of the sector.

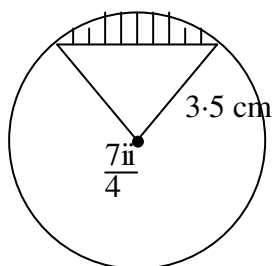
Question 43. (2+3+3=8 MARKS)

Find the area of the following :

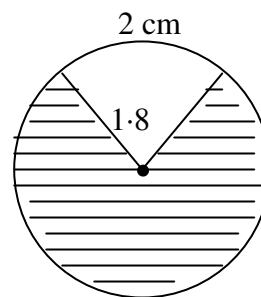
a)



b)



c)



Question 44. (4 MARKS)

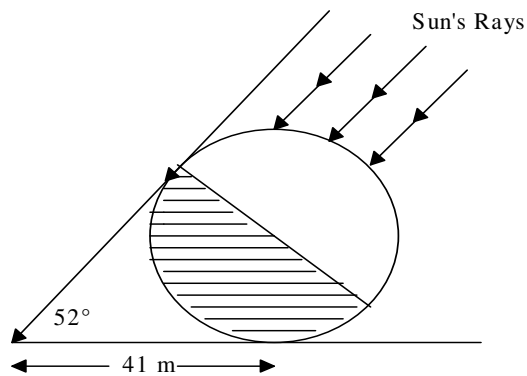
From a point on top of a cliff 150 m above sea level, an observer notices that the angle of depression of a boat A, on a bearing of $S 15^\circ W$ is 35° and that of a second boat B, on a bearing $E 38^\circ S$ is 22° . Find the distance between the two boats.

Question 45. (4 MARKS)

At a point A, the top of a tower T has an angle of elevation of 25° . On walking 100 m closer towards the base of the tower, the angle of elevation is now 40° . Find the height of the tower.

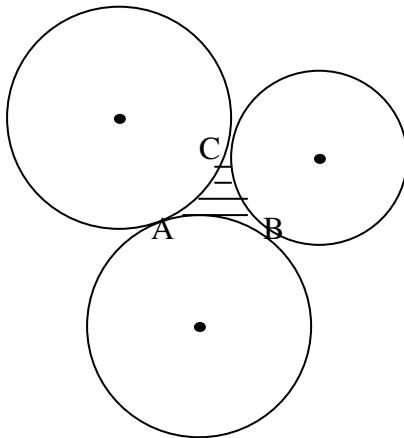
Question 46. (6 MARKS)

A spherical tank rests on level ground and casts a shadow 41 m long, from the point of contact of the sphere with the ground. The angle of elevation of the sun is 52° . Determine the radius of the tank to the nearest metre. How far above the ground is the highest point shaded on the tank.



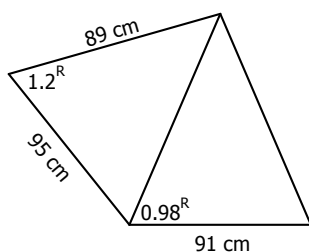
Question 47. (5 MARKS)

Three circles with radii 50, 50 and 30 cm are tangent to each other at the points A, B, and C. Find, correct to 1 decimal place, the area of the curvilinear triangle ABC as shaded in the diagram below.



Question 48.

Cindy has a oddly shaped room that she wants to cover in linoleum. Calculate the amount of linoleum that she will need.

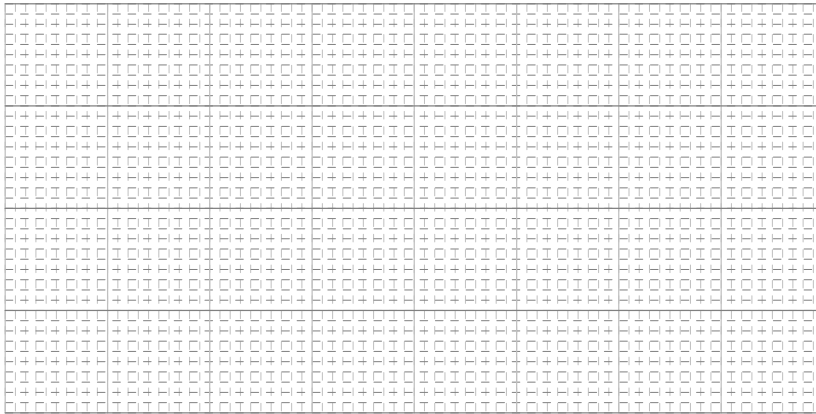


Ch3: Vectors (basic ideas), Ch 4: Vectors (in component form) and Ch 7: Relative displacement and Relative velocity

Question 49. [3, 2, 4 and 2 = 11 marks]

The position vectors of points A and B are $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ respectively.

- (a) Plot these points and determine \overrightarrow{AB} in component form.

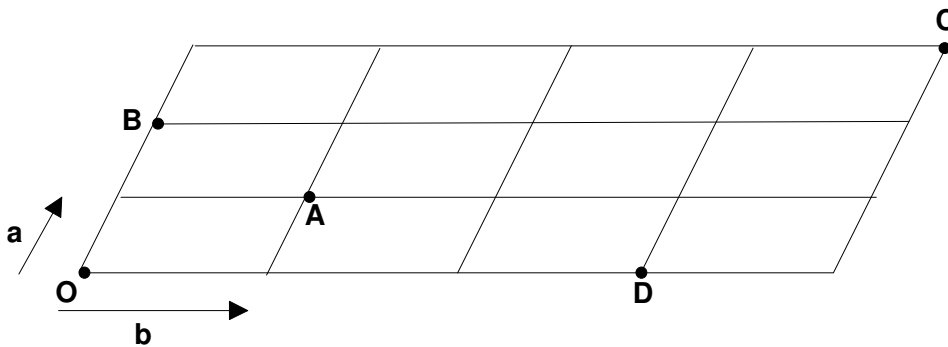


- (b) Find a unit vector in the direction of \overrightarrow{AB} .
- (c) Find the exact magnitude of the vector $\overrightarrow{AB} + \frac{3}{2}\overrightarrow{OB}$.
- (d) If C has co-ordinates (6, y), find y given that the position vector of C is parallel to \overrightarrow{AB} .

Question 50. [1, 1, 1 and 1 = 4 marks]

For the diagram below, express the following vectors in terms of **a** and **b**:

- (a) $\overrightarrow{OA} =$ (b) $\overrightarrow{AB} =$
- (c) $\overrightarrow{BC} =$ (d) $\overrightarrow{CD} =$



Question 51. [1, 2, 5 and 3 = 11 marks]

The position vectors \mathbf{r}_A and \mathbf{r}_B of two ships A and B and their respective velocity vectors \mathbf{v}_A and \mathbf{v}_B are as follows:

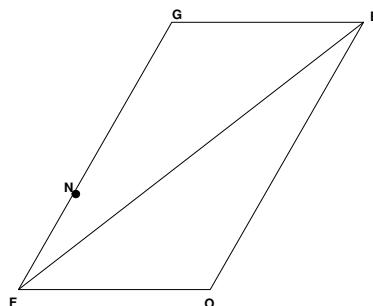
$$\mathbf{r}_A = (11\mathbf{i} + 4\mathbf{j}) \text{ km} \quad (\text{at 11.30am}) \quad \mathbf{v}_A = (2\mathbf{i} - 16\mathbf{j}) \text{ km/hr}$$

$$\mathbf{r}_B = (-22\mathbf{i} - 55\mathbf{j}) \text{ km} \quad (\text{at 12 noon}) \quad \mathbf{v}_B = (10\mathbf{i} - 4\mathbf{j}) \text{ km/hr}$$

- (a) Assuming that both ships maintain their velocities, determine the position vector of Ship A at 12 noon.
- (b) Write down the position vector of each ship in terms of t , the number of hours past 12 noon.

Question 52. [5 and 2 = 7 marks]

Parallelogram GEOF is shown in the diagram below. N divides \overline{FG} in the ratio 1:2 (i.e. $NG=2FN$) and similarly T (not shown) divides \overline{FE} in the ratio 1:3.



- (a) If $\overrightarrow{OF} = \mathbf{f}$ and $\overrightarrow{OE} = \mathbf{e}$, express, in terms of \mathbf{f} and \mathbf{e} ,
- \overrightarrow{FE}
 - \overrightarrow{ON}
 - \overrightarrow{OT}
- (b) Hence, show that O, T and N are collinear.

Question 53. [6 marks]

Given \mathbf{m} is a vector of magnitude 5 units and on a bearing of 060° and \mathbf{n} is a vector of magnitude 10 units and on a bearing of 120° . Find the magnitude and direction of $\mathbf{m} + \mathbf{n}$.

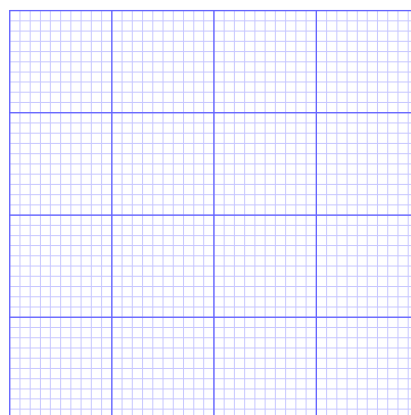
Question 54. [6 marks]

ABCD is a trapezium in which the sides AB and DC are parallel. P and Q are the midpoints of the sides AD and BC respectively. Express the vector \overrightarrow{PQ} in two different ways as the sum of vectors along the sides of the trapezium. Hence prove that \overrightarrow{PQ} is parallel to two of the sides and that its length is $\frac{1}{2} (|\overrightarrow{AB}| + |\overrightarrow{DC}|)$

Question 55. [7 marks]

Using the information below, plot points A to G on the grid.

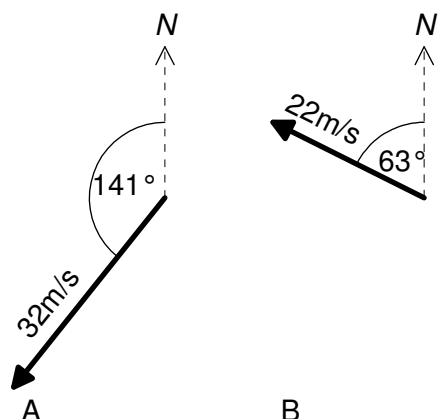
- $\mathbf{r}_A = -4\mathbf{i} + \mathbf{j}$
- $\mathbf{r}_B = 2\mathbf{i} + 6\mathbf{j}$
- $\mathbf{r}_C = -9\mathbf{i} - 9\mathbf{j}$
- $\mathbf{r}_D = -4\mathbf{i} - 2\mathbf{j}$
- $\mathbf{r}_E = 4\mathbf{i} - 6\mathbf{j}$
- $\mathbf{r}_F = \mathbf{r}_B$
- $\mathbf{r}_G = \mathbf{r}_B + \mathbf{r}_A$

**Question 56. [3 marks]**

Prove $\mathbf{r}_C - \mathbf{r}_B = \mathbf{r}_A$

Question 57. [5 marks]

Use a scale drawing to determine ${}_B\mathbf{v}_A$ giving your answer as magnitude and direction.

**Question 58. [6 marks]**

Use trigonometry to confirm your answer to the previous question.

Question 59. [5 + 4 = 9 marks]

A sailing boat is being steered on a course of 330° while it is powered by an 8.8 km/h breeze (as measured by the instruments aboard). Meanwhile, the instruments on a nearby stationary buoy measure the wind speed at 6.6 km/h and its direction as 025° . (Note: wind directions give the direction the wind is blowing from, not the direction it is blowing towards.)

Determine:

- The speed of the boat.
- The direction of the wind as perceived by an observer on the boat.

Ch 6: Indices and logarithms**Question 60. [3,4=7 marks]**

Find, showing all working and giving your answer to 3 significant figures where appropriate, the value of x for which

- $3^x = 5$
- $\log_2(2x + 1) - \log_2 x = 2$

Question 61. [4 marks]

Prove that the value of x that satisfies

$$2\log_2 x + \log_2(x - 1) = 1 + \log_2(5x + 4)$$

is a solution to the equation

$$x^3 - x^2 - 10x - 8 = 0$$

Question 62. [2, 2 mark]

Solve using logarithms the following:

a) $5^{x-2} = 29$

b) $4^{3x} = 5^{x-1}$

Question 63. [3 marks]

Simplify $\frac{3^{2x+1} - 9}{3^{2x-1} - 1}$

Question 64. [2, 2 marks]

Evaluate

a) $\log_3 27$

b) $\log_z z^4$

Question 65. (2,2,2,3,3 marks)

The population of Japan at the start of 2001, was 32 million people and increasing at a rate of 2%. The population (in millions) “ t ” years after 1st January 2001 can be modelled by the equation.

$$P_J = 32e^{0.02t}$$

- a) Find the expected population at the start of 2013 to the nearest hundred.
- b) Find the increase in population from the start of 2002 to the start of 2003

Another country, East Timor, had a population of 60 million people at the start of 2001, but it was decreasing at a rate of 2.5%.

- c) Write an equation for P_E the population (in millions) of East Timor t years after 2001.
- d) In which year will the population be less than 30 million?
- e) In which year will the population of Japan and East Timor be the same?

Question 66. (3,3,marks)

Find y in terms of x :

a) $\log y = 3 \log x - \log x^2$

b) $2 \log_5 y = \log_5 4x^2 + 2 \log_5 x$

Question 67. (2,3,3,marks)

A disease is spreading in Australia. The first day the disease was identified, 150 people were diagnosed as having it. The following day another 162 people were diagnosed as having the disease. On the third day of this epidemic a further 175 people were diagnosed positive and another 189 on the fourth day. Assume the trend continues.

- a) Find the number of people diagnosed on the 10th day.
- b) How many days until 1000 are diagnosed as positive in a single day?

Ch 8: Functions

Question 68. (12 marks)

Given $f(x) = \frac{3}{x}$, $g(x) = x^3$, $h(x) = \sin x$,

Find the following:

- (a) $f \circ g(x)$
- (b) $g(h(t))$
- (c) $f \circ g \circ h\left(\frac{-\pi}{2}\right)$
- (d) $h \circ g \circ f(3)$
- (e) $(f \circ g)^{-1}$

Question 69. (4 marks)

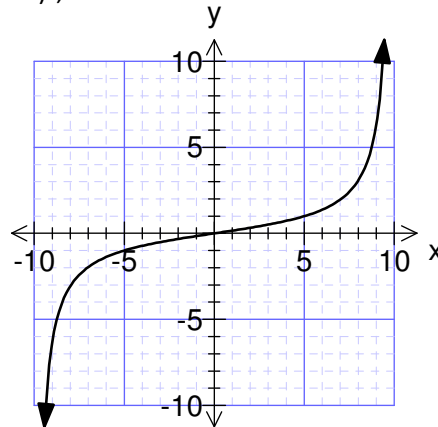
$$h(x) = \sqrt{x-1}, \quad \{x : x \in \mathbf{R}, x \geq 1\}$$

$$k(x) = x^2, \quad \{x : x \in \mathbf{R}, x \geq 0\}$$

What is the natural domain of $h \circ k(x)$? Explain your answer.

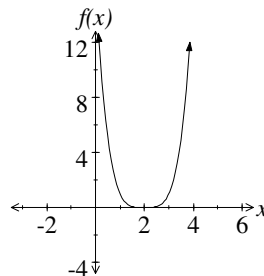
Question 70. (3,4=7 marks)

- (a) Find f^{-1} for the function f where $f(x) = 6 - (x-2)^2$, $x \geq 2$.
- (b) Sketch $g^{-1}(x)$ for $g(x)$ shown:



Question 71. (2, = 6 marks)

The graph of a function $f(x)$ is given below.



- (a) Verify that $f^{-1}(x)$ is not a function for the natural domain of $f(x)$.
- (b) Find the largest possible domain for f such that f^{-1} is a function.
- (c) State the domain and range of $f^{-1}(x)$ which corresponds to the restricted domain of $f(x)$ in (b).

Ch 9: Polar Coordinates

Question 72. (2+2=4 marks) [Simple; Resource free]

- A and B have polar coordinates $(1, \alpha)$ and $(3, \beta)$ respectively. Give the coordinates of A and B in Cartesian form.
- C and D have Cartesian coordinates $(1, -1)$ and $(-\sqrt{3}, 1)$ respectively. Give the coordinates of C and D in Polar form.

Question 73. [4 marks] [Resource rich]

Determine the straight line distance between points A and B where A has Cartesian coordinates $(5, -2)$ and B has polar coordinates $(5, -2^{\text{RADIANS}})$.

Question 74. [4 marks] [Simple; either resource rich or resource free]

Plot the following polar coordinates on the polar grid:

- $(2, \frac{\pi}{3})$
- $(3, -\frac{5\pi}{6})$
- $(4, \frac{3\pi}{4})$
- $(-1, \frac{5\pi}{3})$

